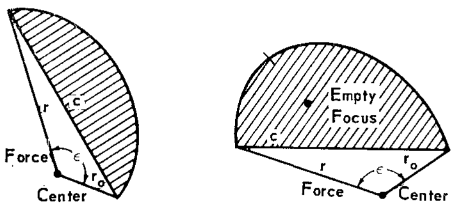

Fig. 1 Geometry of elliptic trajectories.

and

$$\frac{dN}{dz} = \frac{1}{|z|2^{1/2}}\left\{\frac{k}{(1-z)^{1/2}} - \frac{w^3}{(1-w^2z)^{1/2}} - \frac{3N}{2^{1/2}}\right\} \quad \text{(8b)}$$

In these equations, for hyperbolic orbits, f = arcsinh, $k = +1$, and for elliptic orbits, f = arcsin, $k = \pm 1$.

These equations are free from ambiguity if it is agreed that $k = +1$ when the empty focus falls outside the area enclosed by the chord and the trajectory and $k = -1$ if it falls inside of it. Also w is positive if the central angle is less than π and negative if greater than π (see Fig. 1). Finally, Eq. (3) with the new notation becomes

$$N = (2^{1/2}/3)(1 - w^3) \quad \text{(9)}$$

For $z \backsimeq 0$, Eqs. (8) exhibit undeterminancy problems that can be eliminated by broadening the region of validity of Eq. (9).

From Eq. (8a) with $k = +1$, and permitting $z^{1/2}$ to take on imaginary values, one gets

$$\frac{d(z^{3/2}N)}{dz} = \frac{1}{2^{1/2}}\left\{\frac{z^{1/2}}{(1-z)^{1/2}} - \frac{w^3 z^{1/2}}{(1-w^2z)^{1/2}}\right\} \quad \text{(10)}$$

Expanding Eq. (10) around $z = 0$ by

$$(1 - z)^{1/2} = \sum_{n=0}^{\infty} A_n z^n$$

$$\text{where } A_0 = 1 \qquad A_n = \frac{1\cdot 3\cdot 5 \ldots (2n-1)}{2\cdot 4\cdot 6 \ldots (2n)} \quad \text{(11)}$$

it becomes

$$\frac{d(z^{3/2}N)}{dz} = \frac{z^{1/2}}{2^{1/2}}\left\{\sum_{n=0}^{\infty} A_n z^n - w^3 \sum_{n=0}^{\infty} A_n w^{2n} z^n\right\} \quad \text{(12)}$$

which can be integrated to yield

$$z^{3/2}N = \frac{1}{2^{1/2}}\left\{\sum_{n=0}^{\infty} \frac{A_n}{n + 3/2}(1 - w^{2n+3})z^{n+3/2}\right\} + C \quad \text{(13)}$$

The constant of integration for $z = 0$ is obtained as $C = 0$. Then dividing by $z^{3/2}$ the result is (see also Ref. 2)

$$N = 2^{1/2} \sum_{n=0}^{\infty} \frac{A_n}{2n+3}(1 - w^{2n+3})z^n \quad \text{(14a)}$$

The derivative of this equation is

$$\frac{dN}{dz} = 2^{1/2} \sum_{n=1}^{\infty} \frac{nA_n}{2n+3}(1 - w^{2n+3})z^{n-1} \quad \text{(14b)}$$

Now Eq. (14) is valid between $z = -1$ and $+1$, where the power series converges. Note, however, that near the limits the convergence is very slow.

Finally, the question of which equation to use and with what value of z to begin the iteration emerges. To answer this question, three numbers must be calculated which are related to $z = -1, 0, +1$:

$$N_{-1} = 1 - (1/2^{1/2})\{(\sinh^{-1}1 - \sinh^{-1}w) + w(1 + w^2)^{1/2}\} \quad \text{(15)}$$

$$N_0 = (2^{1/2}/3)(1 - w^3) \quad \text{(16)}$$

$$N_{+1} = (1/2^{1/2})\{\pi/2 - \sin^{-1}w + w(1 - w^2)^{1/2}\} \quad \text{(17)}$$

Defining $\Delta = (N_0 - N_{-1})$, the following trouble-free regimes and initial z values can be recommended:

1) $N < \left[N_{-1} + \left(\frac{\Delta}{4}\right)\right]$, use the hyperbolic form of Eqs. (8) with $z_i = -1$ and $k = +1$.

2) $\left[N_{-1} + \left(\frac{\Delta}{4}\right)\right] \leq N < \left(N_0 + \Delta\right)$, use the parabolic form of Eqs. (14) with $z_i = 0$

3) $(N_0 + \Delta) \leq N < N_{+1}$, use the elliptic form of Eqs. (8), with $z_i = 0.75$ $k = +1$

4) $N_{+1} \leq N$, use the elliptic form of Eqs. (8), with $z_i = 0.75$ and $k = -1$ and m according to the problem.

References

[1] Battin, R. H., *Astronautical Guidance* (McGraw-Hill Book Co., Inc., New York, 1964).

[2] Gedeon, G. S., "Lambertian mechanics," *Proceedings of the XII International Astronautical Congress* (Springer-Verlag, Wien, and Academic Press Inc., New York, 1963).

Determination of Mass Eroded from Pulsed Plasma Accelerator Electrodes

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A NOVEL method has been devised for the capture of plasma discharge products for subsequent determination of the eroded electrode mass. The technique also permits simultaneous measurement of the plasma linear momentum for the accelerator considered.

The apparatus consisted of a ballistic pendulum with a cylindrical bob, the axis of which was colinear with the accelerator axis. The pendulum bob was lined with chemically pure filter paper and completely surrounded the discharge zone, including the electrodes. The accelerator consisted of two parallel cylindrical 0.006-m-diam electrodes, 0.2 m in length, spaced on 0.025 m centers. The electrodes were connected through a coaxial spark gap switch to a 6.4-μf capacitor charged to 15 kv. Propellant materials were 1-mil-diam silver wires 0.019 m in length. All experiments were carried out at pressures of 5×10^{-5} torr or less.

The propellant mass accelerated in a typical discharge was on the order of 10^{-7} kg. Since it was assumed that the eroded mass was only a small fraction of the propellant mass (this assumption was subsequently confirmed by the experiment), samples were prepared by accumulating the discharge products of ten successive shots, and an average value of the eroded mass determined therefrom.

A trend of increasing momentum with accumulated firings had been observed[1] when the electrodes were not cleaned and polished between firings. After about 30 shots, the momentum stabilized at a value approximately 10% above the initial value. In order to determine if a correlation existed between erosion and momentum change, deposits were collected for

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the groups consisting of shots 1–10 and 31–40. The latter group was chosen in the range where performance was essentially constant.

Molybdenum and stainless-steel electrode materials were selected for the erosion investigation, since their performance represented the extremes for materials previously considered. Molybdenum captured on the filter paper was prepared by destroying the paper with nitric and perchloric acids and evaporating the resulting solution to dryness. The salts were then dissolved in dilute perchloric acid. One-fifth of the sample solution was reserved for the determination of silver, the propellant material, whereas the remainder of the solution was used for the photometric determination of the mass of molybdenum.

The low concentrations of the constituent elements (iron, chromium, and nickel) of Stainless Steel No. 304 necessitated the development of a new method[2] for the mass determination. The sample was prepared with nitric and perchloric acid as in the case of molybdenum. The silver present was then precipitated as the chloride and removed by filtration. The sample was then deposited as a spot on chromatographic

Table 1 Summary of electrode erosion measurements

Sample designation	Molybdenum electrodes, μg Mo	Stainless-steel electrodes, μg Fe, Ni, and Cr
Shots 1–10	61.5	16.8
Shots 31–40	25.4	5.1

paper using the ring oven technique. A series of standard spots was prepared by using a standard solution of National Bureau of Standards (NBS) Steel 101E, which corresponds to Stainless Steel No. 304. The standard spots were used to provide the calibration curve for the x-ray fluorescence method, and the amounts of iron, chromium, and nickel in the unknown were determined after measuring their x-ray fluorescence intensity.

The portions of the various samples reserved for determination of silver were evaporated to dryness. The mass of silver present was determined by a novel polarographic method developed by Cave and Hume.[3] The silver on the stainless-steel electrodes was removed with dilute nitric acid, and the mass determined polarographically. The mass of silver on the molybdenum electrodes could not be determined because of unexplained difficulty with the polarographic method.

The results of the measurements are summarized in Table 1. A typical example of results obtained with molybdenum electrodes during the first ten shots indicates eroded mass equal to 5.2% of the initial propellant mass. Discharge conditioning of the electrodes results in further reduction of the erosion, which approaches a value equal to 2% of the initial propellant mass. Erosion is still further reduced in the case of the stainless-steel electrodes, amounting to only 0.4% of the propellant mass with conditioned electrodes, although the over-all performance of this material was considerably below that of molybdenum from the standpoint of momentum production. The inferior performance of the stainless-steel electrodes was probably due to their relatively high resistivity.

A check on the validity of the present measurements was provided by measuring the silver propellant, the initial mass of which was known. Significantly, over 97% of this mass was accounted for by the measurement techniques. A further check in the case of stainless-steel electrodes was provided by comparing the percentages of constituent elements of the sample to the specification for AISI (American Iron and Steel Institute) type 304 stainless steel. These percentages were found to fall within the specified limits. The standard deviation was 10% for the stainless-steel analysis and 2% for the molybdenum determination. On these bases the measurements are accepted with a high degree of confidence.

References

[1] Rosebrock, T. L., Clingman, D. L., and Gubbins, D. G., "Pulsed plasma acceleration—final report on contract AF 49(638)-864," General Motors Corp., Allison Div., Rept. EDR 3255 (April 1963).

[2] Cooper, M. D., private communication, General Motors Research Labs., Chemistry Dept. (April 4, 1963).

[3] Cave, G. C. B. and Hume, D. N., "Polarographic determination of low concentrations of silver," Anal. Chem. 24, 588 (1952).

Heating of the Cavity Inside a Spherical Shell Satellite

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Introduction

The satellite considered herein consists of a conducting spherical shell containing inside an ideal gas. It is the purpose of this paper to determine the temperature distribution both in the spherical shell and in the cavity under the boundary condition of a given heat flux applied to the outer surface. This heat flux $Q_1(\theta, t)$ may vary both with time and spherical coordinate θ. Two possible boundary conditions at the inner surface of the shell are considered: 1) the amount of heat radiated from the inner surface to the enclosed gas is proportional to the temperature gradient, and 2) the temperature at the inner surface, and also of the enclosed gas, is a prescribed function $G(\theta, t)$ of t and θ.

An asymptotic solution valid for small time has been obtained for the first problem. The solution to the second problem converges more rapidly than Warren's solution particularly near the inner boundary and for small times.

No account has been taken of the temperature variation of the thermal parameters of the material of the shell. The solutions given hold for both an aerodynamic or radiation heat flux, $Q(\theta, t)$ (with ϕ-wise symmetry). This work can be readily extended to the case with ϕ-wise heat flux variations.

Radiation Law at the Inner Surface

Boundary conditions

We assume that the heat flux applied to the outer surface of the shell, which we denote by $Q_1(\theta, t)$ heat units per unit time per unit surface area, can be expanded as a Fourier-Legendre series:

$$\frac{Q_1(\theta, t)}{K} = Q(\theta, t) = \sum_{n=0}^{\infty} q_n(t)P_n(\cos\theta) \text{ for } t > 0 \tag{1}$$

This series may be finite (if $q_n \equiv 0$ for $n \geqslant N_0$) or infinite; $P_n(\cos\theta)$ is the standard Legendre polynomial of degree n and argument $\cos\theta(= \mu)$, and K is the thermal conductivity. The boundary condition on the outer surface $r = a$ then becomes

$$\left[\frac{\partial T}{\partial r}\right]_{r=a} = Q(\theta, t) = \sum_{n=0}^{\infty} q_n(t)P_n(\mu) \tag{2}$$

where $T(r, \theta, t)$ denotes the temperature in spherical polar coordinates (r, θ, ϕ) whose origin coincides with the center of the spherical shell. At the inner surface $r = b(< a)$, we assume that the radiation condition can be expressed in the Newtonian form

$$\left[\frac{\partial T}{\partial r}\right]_{r=b} = H(T - T_i), \tag{3}$$

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